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EFFECT OF UNSTEADY NATURAL CONVECTION ON THE STRUCTURE OF A LIQUID FLOW IN A HORIZONTAL MIXING CHAMBER

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Results are presented of a numerical and experimental investigation of the effect of natural convection on the structure of a liquid flow in a horizontal mixing chamber with changes in the temperature of the liquid at the inlet.

The study of the effect of natural convection on liquid flow structure, particularly in nuclear power reactor elements with large volumes of coolant, is important to both engineering and science. A characteristic feature of unsteady natural convection due to temperature irregularities and transitional modes of operation is the possibility of stratification of the coolant, with the formation of extensive stagnant zones [1, 2].

In the present work, a horizontal chamber was used to conduct an experimental and numerical study of the structure of liquid flow with a sharp change in temperature at the inlet. The experimental unit was made of organic glass in the form of two rectangular chambers (cross sections of 0.05×0.08 m and 0.05×0.105 m; lengths of 0.24 and 0.15 m, respectively). The chambers were joined at a 90° angle by means of a hydraulic grate installed at the outlet of the horizontal (working chamber). Heaters installed in front of the inlet chamber changed the temperature of the working liquid within the range 15-60°C. The temperature of the distilled water was measured over the height of the working section at a distance of 0.12 m from the outlet, as well as at the inlet and outlet of the horizontal chamber, by Chromel—Copel microthermocouples. The liquid was tinted to make it visible in the chamber.

Figure 1a–c gives a qualitative picture of the characteristic structure of the liquid flow for a transitional (transient) regime, with a decrease in the temperature of the liquid at the inlet and Froude numbers $Fr < 1$. As it enters, the cold liquid occupies the bottom portion of the chamber, and the warmer liquid initially forms a stagnant zone in the top region of the chamber. A feature of this flow regime is that liquid with a temperature $\sim t_0$ is drawn from the chamber outlet into the top, heated region (Fig. 1c). The heated region is relatively stable with respect to connective perturbation from the side of the moving flow. The top region becomes smaller with time and the bottom region increases in size. Temperature is equalized throughout the chamber volume as a result of heat conduction and convection.

Figure 1d–f shows a transitional regime with an increasing liquid temperature at the inlet for $Fr < 1$. Stratification of the liquid flow here is characterized by a cold stagnant zone in the lower region of the chamber. The warmer liquid occupies the top region of the chamber and moves toward the outlet.

Figure 2 shows a temperature profile through the height of the working chamber at a distance of 0.12 m from the outlet for different moments of time after the temperature of the liquid at the inlet in the working section was reduced. The liquid is stratified with respect to temperature through the chamber height, the temperature drop between the top and bottom regions reaching a maximum $\sim(t_0 - t_{in})$ and then gradually decreasing. The liquid is isothermal both in the top, stagnant region of the mixing chamber and in the bottom, convective region. With time, a temperature profile characteristic of heat transfer by conduction is established in the stagnant region. Nearly all of the temperature drop between the regions is concentrated in a narrow boundary layer. Convective perturbations from the side of the moving liquid cannot overcome the jump in density at the interface of the regions, so

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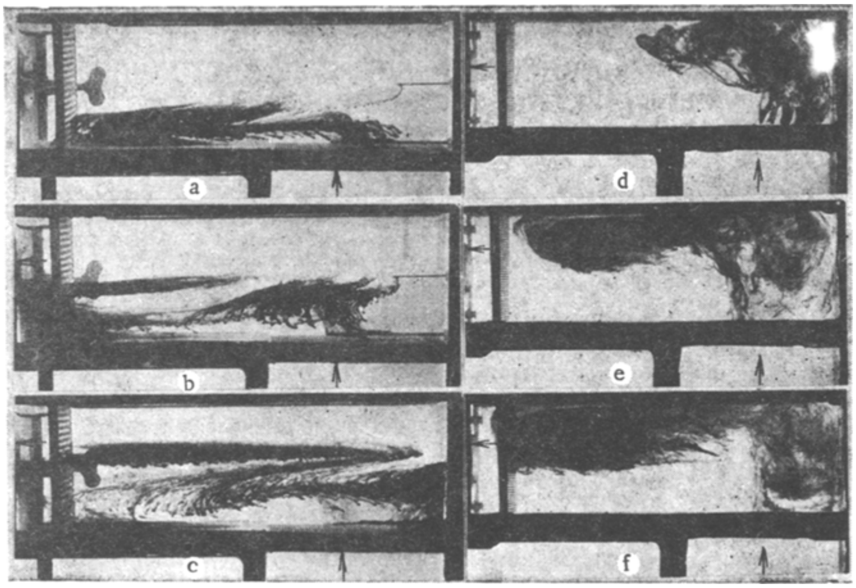


Fig. 1. Character of liquid flow in mixing chamber. Regime with decreasing liquid temperature at the inlet: a) 20; b) 40; c) 60 sec; regime with increasing liquid temperature at the inlet: d) 10; e) 20; f) 30 sec.

that heat exchange between the latter occurs mainly by conduction. The heat transferred from the stagnant region by the molecular method is quickly eroded by turbulent eddies in the convective region. Thus, the velocity gradient between the regions automatically maintains an interface of minimum thickness with a large temperature gradient. The interface may be deformed by convective perturbations from the side of the moving liquid, and individual sections of the interface may undergo random oscillations. The thermocouple installed at the interface records the random low-frequency oscillations in temperature that have a high amplitude, equal to half the temperature drop between the regions.

A mathematical model of the channel can be represented by two horizontal plates separated by a distance H, with the liquid being admitted through a circular hole in the bottom plate. We conducted numerical investigations of the unsteady temperature and velocity fields. The system of equations in the Boussinesq approximation describing the process of laminar mixed forced and natural convection in terms of the variables vorticity–stream function has the form

$$\frac{\partial\omega}{\partial(\mathrm{Ho})}+\frac{1}{\xi}\frac{\partial\Psi}{\partial\xi}\frac{\partial\omega}{\partial X}-\frac{1}{\xi}\frac{\partial\Psi}{\partial X}\frac{\partial\omega}{\partial\xi}=\frac{1}{\mathrm{Fr}}\frac{1}{\xi}\frac{\partial\Theta}{\partial\xi}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial\xi^2}+\frac{3}{\xi}\frac{\partial\omega}{\partial\xi}+\frac{\partial^2\omega}{\partial X^2}\right),$$

$$\frac{\partial^2\Psi}{\partial X^2}-\frac{1}{\xi}\frac{\partial\Psi}{\partial\xi}+\frac{\partial^2\Psi}{\partial\xi^2}=\omega\xi^2, \tag{1}$$

$$\frac{\partial\Theta}{\partial(\mathrm{Ho})}+\frac{1}{\xi}\frac{\partial\Psi}{\partial\xi}\frac{\partial\Theta}{\partial X}-\frac{1}{\xi}\frac{\partial\Psi}{\partial X}\frac{\partial\Theta}{\partial\xi}=\frac{1}{\mathrm{Pe}}\left(\frac{\partial^2\Theta}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\Theta}{\partial\xi}+\frac{\partial^2\Theta}{\partial X^2}\right),$$

where

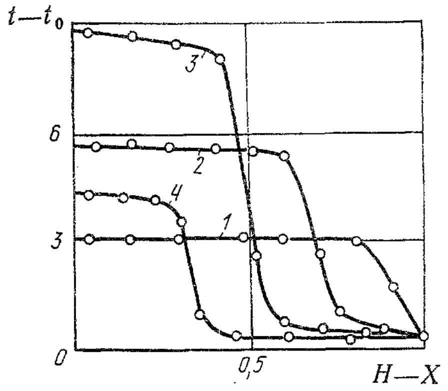
$$\omega\xi=\frac{\partial U}{\partial\xi}-\frac{\partial V}{\partial X};\quad \frac{\partial\Psi}{\partial X}=-\xi V;\quad \frac{\partial\Psi}{\partial\xi}=\xi U;$$

$|t_{in}-t_0|$, v_0, R, R/v_0 are the temperature, velocity, length, and time scales, respectively

The identity conditions for the system (1) were assigned in the form: on the symmetry axis ($\xi=0$, $0\le X\le H$)

$$\frac{\partial\Theta}{\partial\xi}=0,\ \omega=0,\ \Psi=0, \tag{2}$$

at the inlet ($X=0$, $0\le\xi\le 1$)



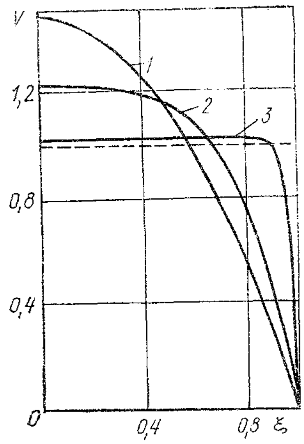


Fig. 2 Fig. 3

Fig. 2. Temperature profiles through height of mixing chamber: 1) 0.05 h; 2) 0.10; 3) 0.15; 4) 0.50 h. Δt, °C.

Fig. 3. Velocity distribution in a porous body: 1) $\zeta = 1$; 2) 26; 3) 1500.

$$\Theta = \pm 1,\ \omega = 0,\ \Psi = \frac{1}{2}\xi^2, \tag{3}$$

on the walls (the conditions for vorticity ω are obtained from the equation for the stream function Ψ)

$$\begin{gathered} \frac{\partial\Theta}{\partial X} = 0,\ \omega = \frac{1}{\xi^2}\frac{\partial^2\Psi}{\partial X^2}, \\ \Psi = \frac{1}{2}\ (X = 0,\ \xi > 1), \\ \Psi = 0\ (X = H,\ \xi \geqslant 0). \end{gathered} \tag{4}$$

In solving the Navier—Stokes equations (1), a certain ambiguity arises in connection with formulating the boundary conditions at the outlet. If the investigated channel is sufficiently long, then we may assign at the interface a velocity profile corresponding to steady flow. Since it is the initial section of the flow that is of interest in most cases, it is uneconomical to obtain a numerical solution for the entire region. The formulation of such so-called "mild" boundary conditions allows us to reduce the size of the calculated region. It may be reasoned that the fewest limitations will be placed on the flow by outlet boundary conditions in the form of boundary-layer equations [3], the derivation of which presumes the triviality of the longitudinal gradients compared to the transverse gradients.

In actual apparatus, coolant leaves the mixing chambers through equipment with different hydraulic resistances. In the case where the pressure gradient in the mixing chamber is substantially less than the pressure gradient in the power channel (large hydraulic resistance), we may use "severe" boundary conditions — a shock velocity profile.

It was shown in [4] that, to a substantial degree, flow in tube bundles is determined by the inertia of the coolant, i.e., a direct connection should exist between the structure of the flows in the mixing channel and the power channel. It is therefore of interest to obtain outlet boundary conditions that consider the resistance of the hydraulic grate. Such an attempt was made in [5], where "mild" boundary conditions that were obtained were, with an increase in the hydraulic resistance of the grate to infinity, converted into a shock velocity profile. However, such a limiting conversion to boundary-layer boundary conditions is lacking with a decrease in outlet resistance [3].

Using a model of a porous body for the outlet hydraulic grate, we will assume that, apart from shear stresses due to the viscosity of the coolant, a force proportional to the velocity of the liquid flow is acting on an elementary volume of the latter (Darcy's law

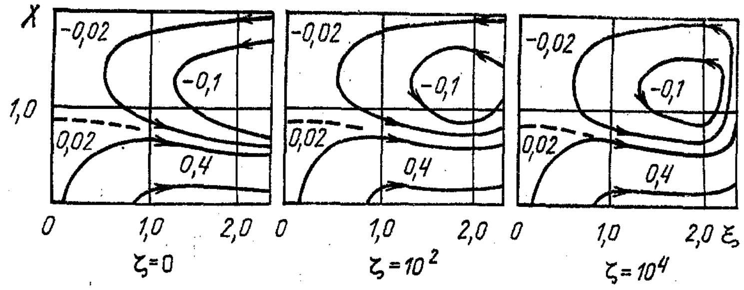


Fig. 4. Effect of boundary conditions on flow structure in a mixing chamber at Ho = 3.

[4]). Assuming that the porous body is isotropic, we obtain the following form for the additive term in the Navier—Stokes equations

$$-K^* \frac{\nu}{d^2_{\text{equ}}} \mathbf{W}, \tag{5}$$

where K* is a constant coefficient.

Changing over to the equation of motion in the variables vorticity—stream function and ignoring the second derivatives with respect to flow direction, we obtain "mild" boundary conditions at the outlet (in the general case, the coefficient of hydraulic resistance ζ may be considered a function of the coordinate X):

$$\frac{\partial \omega}{\partial (\mathrm{Ho})} + \frac{1}{\xi} \frac{\partial \Psi}{\partial \xi} \frac{\partial \omega}{\partial X} - \frac{1}{\xi} \frac{\partial \Psi}{\partial X} \frac{\partial \omega}{\partial \xi} = \frac{1}{\xi \mathrm{Fr}} \frac{\partial \Theta}{\partial \xi} + \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 \omega}{\partial X^2} - \zeta\omega - \frac{\partial \Psi}{\partial X} \frac{\partial \zeta}{\partial X} \right),$$
$$\frac{\partial^2 \Psi}{\partial X^2} = \omega \xi^2, \tag{6}$$
$$\frac{\partial \Theta}{\partial (\mathrm{Ho})} + \frac{1}{\xi} \frac{\partial \Psi}{\partial \xi} \frac{\partial \Theta}{\partial X} - \frac{1}{\xi} \frac{\partial \Psi}{\partial X} \frac{\partial \Theta}{\partial \xi} = \frac{1}{\mathrm{Pe}} \frac{\partial^2 \Theta}{\partial X^2},$$

where $\zeta = K^*(R^2/d^2_{\text{equ}})$.

To illustrate the flow in the porous body represented by the mathematical model (5), let us examine an analytical solution for a steady plane liquid flow. The equation of motion, without the effect of natural convection, and the boundary conditions have the form

$$\frac{1}{\rho} \frac{dP}{dx} = \nu \frac{d^2 v}{dy^2} - K^* \frac{\nu}{d^2_{\text{equ}}} v, \tag{7}$$

$$\left. \frac{dv}{dy} \right|_{y=0} = 0, \quad v|_{y=R} = 0, \tag{8}$$

where R is half the width of the channel.

Taking the boundary conditions (8) into account, we obtain the solution to Eq. (7)

$$v = \frac{-d_{\text{equ}}}{K^* \rho \nu} \frac{dP}{dx} \left[1 - \frac{\mathrm{ch}(\sqrt{\zeta}\, \xi)}{\mathrm{ch} \sqrt{\zeta}} \right]. \tag{9}$$

Introducing the average velocity of the coolant over the cross section v_0, we write the dimensionless velocity profile in the form

$$V = \left[1 - \frac{\mathrm{ch}(\sqrt{\zeta}\, \xi)}{\mathrm{ch} \sqrt{\zeta}} \right] \Big/ \left(1 - \frac{\mathrm{th} \sqrt{\zeta}}{\sqrt{\zeta}} \right). \tag{10}$$

As $\zeta \to 0$, the velocity distribution (10) changes into a parabolic distribution (steady-state flow between stationary plates)

$$V = 1.5(1 - \xi^2). \tag{11}$$

As $\zeta \to \infty$, the maximum velocity approaches unity, i.e., the velocity profile becomes increasingly filled. The velocity distribution (10) is shown in Fig. 3 for different values of ζ.

The transitional regimes in the mixing channel represented by the system of equations (1) were studied at zero initial conditions

$$\omega_0(X, \xi) = 0, \; \Psi_0(X, \xi) = 0, \; \Theta_0(X, \xi) = 0. \tag{12}$$

In solving problems (1)-(6) numerically, we used an implicit difference scheme with a spatial approximation of second-order accuracy [6] in which the convective terms were represented by the central differences. The second derivative of the stream function in the boundary condition for vorticity (4) was approximated by a formula of second-order accuracy [3].

Let us examine the structure of the flow in the channel at a liquid temperature at the inlet $\Theta = -1$. The relatively cold liquid entering the chamber is reflected from the more heated layers in the top region and moves toward the outlet. A reverse flow of heated liquid arises in the top region of the chamber, the velocity of this flow being determined by the boundary conditions at the outlet from the channel. Figure 4 shows lines of flow in the mixing channel for the moment of time $Ho = 3$ in relation to the coefficient of hydraulic resistance ζ. Even when the value of the coefficient is relatively large, coolant continues to flow into the mixing chamber from the outside.

The appearance of the characteristic layer of heated liquid in the top region of the mixing chamber (the reverse movement of coolant with "mild" boundary conditions, or the large-scale eddy with a shock velocity profile at the chamber outlet) depends on the Froude number, which determines the effect of heat convection forces. For those cases where the temperature at the inlet is equal to $\Theta = 1$, the liquid flow is compressed as it is moved into the top region of the mixing chamber due to buoyancy. The flow is stratified with respect to temperature, and a reverse flow of cold liquid in the bottom region of the channel is formed. Thus, the results of the numerical studies agree with the experimental results. The numerical investigations (for $Fr < 1$) showed that, at relatively low Reynolds numbers, the predominant forces in forming the flow in the channel are natural convective forces. The boundary value of the Reynolds number Re at which the effect of viscous forces may be ignored depends substantially on the height of the chamber. With a decrease in the relative height of the chamber $H = 2$-0.5, Reynolds number similitude begins at $Re \geq 200$-2000, respectively (calculations were performed for $Re > 200$ with an approximation of the differential equations by a monotonic difference scheme of second-order accuracy [7]).

Heat exchange between the stagnant and convective regions of the liquid occurs by conduction, so that the Peclet number — characterizing the contribution of molecular heat transfer — turns out to have a direct effect on the process of temperature equalization in the mixing channel. Thus, the similitudinous conditions of liquid flow with respect to the Reynolds number at relatively low values of this number makes it possible to experimentally model transitional regimes of coolant motion according to the Froude and Peclet numbers. In particular, in this case it is possible to use water in experiments to simulate mixing chambers with a liquid-metal coolant.

Generalizing the results of the completed studies, we should note that the coolant stratification that occurs, e.g., in the mixing chambers of fast power reactors — in which the temperature drop between regions of coolant stratification may exceed 150°C at Froude numbers less than 1 — results in substantial thermal stresses in structural elements. Moreover, temperature stratification of the coolant under transitional and emergency conditions of reactor operation has an appreciable effect on the development of the natural circulation of the coolant in the circuit.

NOTATION

t_0, t_{in}, initial temperature and temperature at the inlet to the channel; Θ, dimensionless temperature; a, heat conductivity; ν, kinematic viscosity; β, coefficient of cubical expansion; ρ, density; P, pressure; g, acceleration due to gravity; d_{equ}, equivalent diameter of porous body; τ, time; v_0, mean velocity at the inlet; X and ξ, dimensionless vertical and radial coordinates; U and V, dimensionless vertical and horizontal components of velocity; H, dimensionless height of channel; R, radius of inlet to channel; **W**, velocity of liquid; Δt, temperature drop along channel height; $Re = v_0 R/\nu$, Reynolds number; $Pe = v_0 R/a$, Peclet number; $Fr = v_0^2/g\beta|t_{in} - t_0|R$, Froude number; $Ho = v_0\tau/R$, homochroneous number.

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STRUCTURE OF FLOWS IN VORTICES

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UDC 532.517.4

The structure of a gradient vortical flow was studied experimentally.

Stationary vortex tubes, characterized by substantial radial changes in velocity, were excited by a vortex generator [1]. The frequency of rotation of the swirler was measured with a tachometer. Velocity and pressure were measured with a five-channel spherical probe which was moved radially and vertically.

Vortex generators are usually characterized by dimensionless parameters [2]. The values of certain of these parameters for the above generator are given in Table 1. Also shown here for the sake of comparison are values for the units in [3].

According to the data in [2], free vortices have sharp velocity gradients at the boundary of the core in those cases where the effective exchange coefficient s and ratio a^*/a are close to unity. The vortex generator we used did not allow us to independently change the velocity of the ascending flow or the angular momentum of the vortex. The parameter s thus changed within very narrow limits, whereas the generators in [3], with constant equipment dimensions, allowed for variation of this parameter. As concerns the parameter a^*/a,

TABLE 1. Dimensionless Parameters Characterizing the Vortices

Parameter	Model: data from [3]	Model: our data	Parameter	Model: data from [3]	Model: our data
Γ_∞, m²/sec	1	1,3	a^*	1,6	2,0
Re, 10^4	5	8,5	a^*/a	0,11	0,9
N, 10^4	7,7	5,1	s	0,3	0,8
a	15	2,3	s^*	5,5	2,0

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